ÉRETTSÉGI VIZSGA • 2013. május 7.

MATEMATIKA ANGOL NYELVEN

EMELT SZINTŰ ÍRÁSBELI ÉRETTSÉGI VIZSGA

JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ

EMBERI ERŐFORRÁSOK MINISZTÉRIUMA

Instructions to examiners

Formal requirements:

1. Mark the paper in **ink, different in colour** from the one used by the candidate. Indicate the errors, incomplete solutions, etc in the conventional way.
2. The first one of the rectangles under each problem shows the maximum attainable score on that problem. The **points** given by the examiner are to be entered **in the rectangle** next to that.
3. **If the solution is perfect,** it is enough to enter the maximum scores in the appropriate rectangles.
4. If the solution is incomplete or incorrect, please indicate the individual **partial scores** in the body of the paper, too.

Assessment of content:

1. The markscheme may contain more than one solution for some of the problems. If the **solution by the candidate is different**, allocate the points by identifying the parts of the solution equivalent to those of the one given in the markscheme.
2. The subtotals in the markscheme can be **further divided**, but the scores awarded should always be whole numbers.
3. If it is clear that the reasoning and the final answer are both correct, you may award the maximum score even if the solution is **less detailed** than the one in the markscheme.
4. If there is a **calculation error** or inaccuracy in the solution, only take off the points for that part where the error is made. If the reasoning remains correct and the error is carried forward, the points for the rest of the solution should be awarded.
5. In the case of a **principal error,** no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the markscheme.) However, if the wrong information obtained owing to the principal error is carried forward to the next section or to the next part of the problem and is used correctly there, the maximum score is due for the next part, provided that the error has not changed the nature of the task to be completed.
6. Where the markscheme shows a **remark** or **unit** in brackets, the solution should be considered complete without that unit as well.
7. If there are more than one different approaches to a problem, **assess only the one indicated by the candidate**.
8. **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
9. **Do not take off points** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
10. **Assess only four out of the five problems in Section II.** The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted in their total score. Should there be a solution to that problem, it does not need to be marked. However, if it is still not clear which problem the candidate does not want to be assessed, assume automatically that it is the last one in the question paper, and do not assess that problem.

I.

1. a)

A number x may only be a solution if $0.5 < x$.	1 point	*This point is also due if the candidate only checks the correct result against the domain.*
Since $0 = \log_{\frac{1}{5}} 1$,	1 point	
and the base $\frac{1}{5}$ logarithm function is strictly decreasing, it follows that $2x - 1 > 1$,	1 point	
that is $x > 1$. (These real numbers agree with the restriction, too.)	1 point	
Total:	**4 points**	

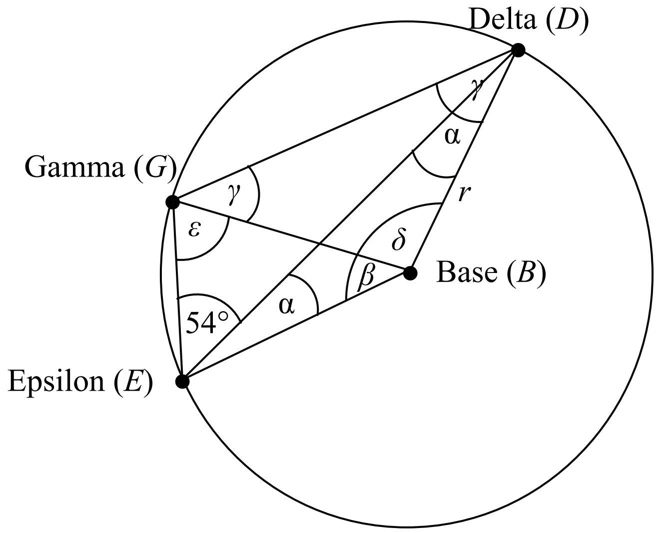
1. b)

$1 = 2^0$, and	1 point	*These 2 points are also due for writing down the inequality without an explanation.*
since the base 2 exponential function is strictly increasing, it follows that $\lvert 2x - 1\rvert - 2 > 0$,	1 point	
that is $\lvert 2x - 1\rvert > 2$. This inequality holds if and only if $2x - 1 > 2$, or	1 point	*These 4 points are also due for reading the solution from a graph (2 points), and checking that those are the exact boundaries (2 points).*
$2x - 1 < -2$.	1 point	
That is, $x > 1.5$, or	1 point	
$x < -0.5$. (The set of solutions is $]-\infty\,;-0.5[\,\cup\,]1.5\,;+\infty[$)	1 point	
Total:	**6 points**	

2. a)

The 1 : 500 000 scale means that 1 cm on the map corresponds to 500 000 cm in reality,	1 point	
which is 5 km.	1 point	
Thus the distance from the base to the wells is $3.5 \cdot 5 = 17.5$ km.	1 point	
Total:	**3 points**	*Award full 3 points for a correct answer without an explanation.*

2. b) Solution 1



With the notations of the diagram, the triangles *EBD*, *BEG* and *BGD* are isosceles, since point *B* is equidistant from the other three points.	1 point	
$\beta+\delta=142°$, $\alpha=\frac{180°-142°}{2}=19°$,	1 point	
$\varepsilon=54°+19°=73°$,	1 point	
$\beta=180°-2\cdot 73°=34°$,	1 point	
$\delta=142°-34°=108°$.	1 point	
For example, from the cosine rule:	1 point	
$GD=\sqrt{17.5^2+17.5^2-2\cdot 17.5^2\cdot\cos 108°}\approx 28.3$ (km),	1 point	
$EG=\sqrt{17.5^2+17.5^2-2\cdot 17.5^2\cdot\cos 34°}\approx 10.2$ (km),	1 point	
$ED=\sqrt{17.5^2+17.5^2-2\cdot 17.5^2\cdot\cos 142°}\approx 33.1$ (km).	1 point	
The total distance covered on Mondays: $BE+EG+GD+DB=17.5+10.2+28.3+17.5=$ $=73.5\approx 74$ km.	1 point	
The total distance covered on Thursdays: $BG+GE+ED+DB=17.5+10.2+33.1+17.5=$ $=78.3\approx 78$ km.	1 point	
Total:	**11 points**	

2. b) Solution 2		
The centre of the circumscribed circle of triangle *EGD* is *B*, and its radius is $r = 17.5$ (km), since that is the distance from *B* to the other three points.	1 point	
The central angle *GBD* is 108°,	1 point	
since it is the double of the 54° angle *GED* on the circumference.	1 point	
The central angle *EBG* is 142° – 108°= 34°.	1 point	
It follows from the relationship of angles at the centre and at the circumference that $\angle EDG = 17°$,	1 point	
and $\angle EGD = 109°$.	1 point	
The sides of triangle *EDG* can be obtained from the formula $a = 2r\sin\alpha$: $GD = 35 \cdot \sin 54° \approx 28.32$ km,	1 point	
$EG = 35 \cdot \sin 17° \approx 10.23$ km,	1 point	
$ED = 35 \cdot \sin 109° \approx 33.09$ km.	1 point	
The total distance covered on Mondays: $BE + EG + GD + DB = 17.5 + 10.23 + 28.32 + 17.5 =$ $= 73.55 \approx 74$ km.	1 point	
The total distance covered on Thursdays: $BG + GE + ED + DB \approx 17.5 + 10.23 + 33.09 + 17.5 =$ $= 78.32 \approx 78$ km.	1 point	
Total:	**11 points**	
With the sides of triangle EGD rounded to integers: DG=28, EG=10; ED=33. Thus the total distance on Mondays is 73 km, and the total distance on Thursday is 78 km. Accept this calculation. too.		

<table>
<tr><td colspan="3">3. a)</td></tr>
<tr><td>In base 3 notation, the digit b of the three-digit number abb_3 may have three different values, and the digit a may have two different values. That makes 6 numbers of the form abb_3.</td><td>1 point</td><td></td></tr>
<tr><td>The six numbers are represented in the table below, in both base 6 and base 10 notations.

<table>
<tr><th>a</th><th>b</th><th>abb_3</th><th>decimal</th></tr>
<tr><td>1</td><td>0</td><td>100</td><td>9</td></tr>
<tr><td>1</td><td>1</td><td>111</td><td>13</td></tr>
<tr><td>1</td><td>2</td><td>122</td><td>17</td></tr>
<tr><td>2</td><td>0</td><td>200</td><td>18</td></tr>
<tr><td>2</td><td>1</td><td>211</td><td>22</td></tr>
<tr><td>2</td><td>2</td><td>222</td><td>26</td></tr>
</table></td><td>3 points</td><td>1 point for at least 8 correct numbers out of 12, 2 points for 11 correct numbers.</td></tr>
<tr><td>The requirements of the problem are met by three numbers:
$200_3=18$, $211_3=22$ and $222_3=26$.</td><td>1 point</td><td></td></tr>
<tr><td align="right">Total:</td><td>5 points</td><td></td></tr>
</table>

3. b) Solution 1		
A five-element set has $2^5 = 32$ subsets.	1 point	
The number of zero-element subsets is 1,	1 point	
and the number of one-element subsets is 5.	1 point	
That is, the number of subsets of at least 2 elements is $32 - 6 = 26$.	1 point	
The product of the elements in a subset is **not** divisible by three if and only if it contains no other elements than 2, 4, or 5.	1 point	
The number of such two-element subsets is $\binom{3}{2} = 3$.	1 point	
and there is 1 such three-element subset.	1 point	
Hence the number of subsets in question is $(26 - 4 =)\, 22$.	1 point	
Total:	**8 points**	

3. b) Solution 2		
Let us count those subsets of at least two elements of the set {2; 3; 4; 5; 6} in which the product of the elements is divisible by three,	1 point	
that is, those that contain an element divisible by 3 (3 or 6).	1 point	
There are 8 subsets that contain 3 but do not contain 6. ($8 = 2^3$, since any subset of 2, 4, 5 may be chosen next to the 3.)	1 point	
These include the one-element set $\{ 3 \}$, so 7 have at least 2 elements.	1 point	
There are 8 subsets that contain 6 but do not contain 3. ($8 = 2^3$, since any subset of 2, 4, 5 may be chosen next to the 6.)	1 point	
These include the one-element set $\{ 6 \}$, so 7 have at least 2 elements.	1 point	
There are also 8 subsets that contain both 3 and 6 ($8 = 2^3$, since any subset of 2, 4, 5 may be chosen again next to the 3 and 6.)	1 point	
Therefore the number of all subsets in question is $(7 + 7 + 8 =)\ 22$.	1 point	
Total:	**8 points**	

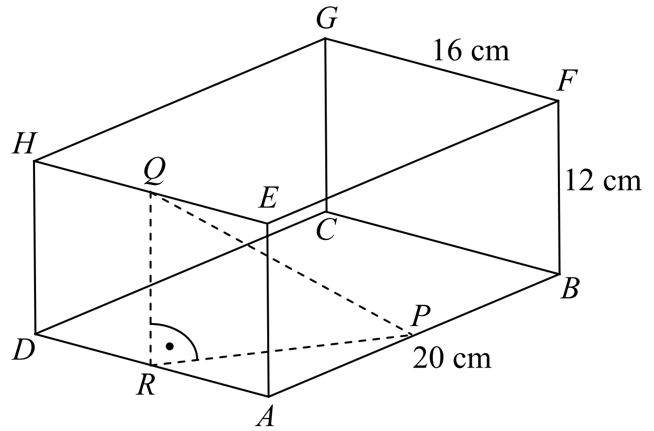
3. b) Solution 3		
The product of the elements in a subset of at least two elements is divisible by 3 if and only if at least one element is divisible by 3 (that is, if the subset contains at least one of 3 and 6).	1 point	
The number of two-element subsets is $\binom{5}{2}=10$.	1 point	
$\binom{3}{2}=3$ of these do not contain either 3 or 6.	1 point	
The number of three-element subsets is $\binom{5}{3}=10$,	1 point	
and only one of these (the set $\{2\,;4\,;5\}$) contains neither 3 nor 6.	1 point	
Subsets of at least four elements all contain at least one of 3 and 6, so they all meet the requirement.	1 point	
The number of subsets of at least four elements is $\left(\binom{5}{4}+\binom{5}{5}=\right)6$.	1 point	
Therefore the number of all subsets in question is 7+9+6=22.	1 point	
Total:	**8 points**	

Remarks.
1. The table below lists all subsets of at least two elements, and those that meet the requirements of the problem.

subset	all	product is divisible by 3
2 elements	$\{2;3\}$, $\{2;4\}$, $\{2;5\}$, $\{2;6\}$, $\{3;4\}$, $\{3;5\}$, $\{3;6\}$, $\{4;5\}$, $\{4;6\}$, $\{5;6\}$.	$\{2;3\}$, $\{2;6\}$, $\{3;4\}$, $\{3;5\}$, $\{3;6\}$, $\{4;6\}$, $\{5;6\}$.
3 elements	$\{2;3;4\}$, $\{2;3;5\}$, $\{2;3;6\}$, $\{2;4;5\}$, $\{2;4;6\}$, $\{2;5;6\}$, $\{3;4;5\}$, $\{3;4;6\}$, $\{3;5;6\}$, $\{4;5;6\}$.	$\{2;3;4\}$, $\{2;3;5\}$, $\{2;3;6\}$, $\{2;4;6\}$, $\{2;5;6\}$, $\{3;4;5\}$, $\{3;4;6\}$, $\{3;5;6\}$, $\{4;5;6\}$.
4 elements	$\{2;3;4;5\}$, $\{2;3;4;6\}$, $\{2;3;5;6\}$,. $\{2;4;5;6\}$, $\{3;4;5;6\}$.	$\{2;3;4;5\}$, $\{2;3;4;6\}$, $\{2;3;5;6\}$, $\{2;4;5;6\}$, $\{3;4;5;6\}$.
5 elements	{2; 3; 4; 5; 6}	{2; 3; 4; 5; 6}

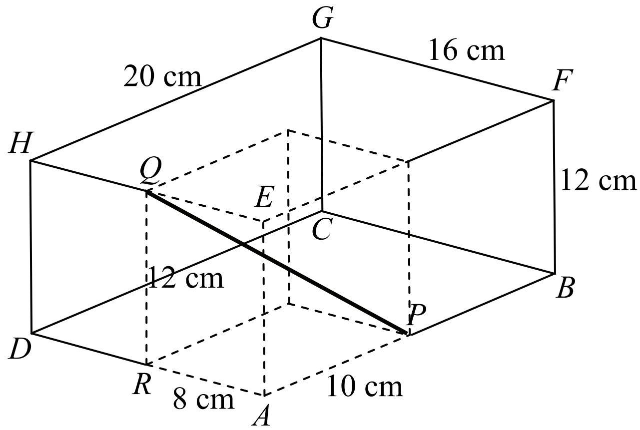
2. It is also accepted if the candidate selects (lists) those subsets that do not meet the requirements of the problem.
3. Award at most 6 points if the candidate only lists the right subsets but does not mention why the list is complete.
4. In the case of a logical sieve applied, the allocation of points is as follows:
"contains 3" + "contains 6" – "contains 3 and 6" *4 points*
$\left(2^4-1\right)+\left(2^4-1\right)-2^3=15+15-8=22$ *4 points*

4. a) Solution 1



Let R be the midpoint of edge AD. Then triangle PRQ has a right angle at R.	1 point	*This point is also due if the idea is only reflected by the solution.*
In the right-angled triangle PAR (with the Pythagorean theorem): $PR^2 = 10^2 + 8^2 (= 164)$.	1 point	
Since $QR = AE = 12$ (cm), with the Pythagorean theorem applied to the right-angled triangle PRQ:	1 point	
$PQ^2 = PR^2 + QR^2 = 12^2 + 10^2 + 8^2$. $PQ = \sqrt{308}$ (≈ 17.55) (cm).	1 point	
Total:	**4 points**	

4. a) Solution 2



PQ is the diagonal of a cuboid one vertex of which is *A*, where the lengths of the three edges are 10 cm, 8 cm and 12 cm.	2 points	
Therefore the length of diagonal *PQ* is $PQ=\sqrt{12^2+10^2+8^2}=\sqrt{304}\ (\approx 17.55)$ (cm).	2 points	*1 point for applying the correct formula of the diagonal, and 1 point for correct calculation.*
Total:	**4 points**	

4. b)

The number of possible selections of edge pairs equals the number of ways to select 2 elements out of 12 different elements, irrespective of the order.	1 point	*Award this point if the solution is correct but no explanation is given.*
The number of different line pairs is $\binom{12}{2}$,	1 point	
which equals $\left(\frac{12\cdot 11}{2}=\right)66$.	1 point	
Total:	**3 points**	

4. c)		
The line of each edge is intersected by 4 other lines, thus the number of intersecting pairs is $\frac{12 \cdot 4}{2} = 24$.	2 points	*1 point is due for finding the right method of counting, even if the correct reasoning is only reflected by the solution.*
The line of each edge is parallel to 3 other lines, thus the number of parallel pairs is $\frac{12 \cdot 3}{2} = 18$.	1 point	
The line of each edge is skew relative to 4 other lines, thus the number of skew pairs is $\frac{12 \cdot 4}{2} = 24$.	1 point	
Total:	**4 points**	
It is possible to calculate two out of the three numbers and obtain the third number by subtraction from the result of question b). Should there be an error in part b) that is carried forward here with no further error made, award full points for part c).		

4. d)		
The distance between two skew lines is the length of the edge lying on the third line that intersects both of them at right angles (normal transversal).	1 point	*This point is also due if there is an annotated diagram with the distances indicated but there is no verbal explanation.*
The distance between lines AE and FG (or BC) is $(EF = AB =)$ 20 cm.	1 point	
The distance between lines AE and HG (or DC) is $(EH = AD =)$ 16 cm.	1 point	
Total:	**3 points**	

II.

5. a) Solution 1		
The common ratio of the geometric progression is a positive number less than 1, therefore the sequence S_n of the sums converges,	1 point	
and its limit is $s = \frac{a_1}{1-q} = \frac{32}{1-\frac{1}{128}} = \frac{4096}{127} (\approx 32.25)$.	2 points	
(Since all terms of the geometric progression are positive, the sequence S_n is increasing, so) $S_n < s = \frac{4096}{127} < 32.5$, therefore the statement is true.	1 point	
Total:	**4 points**	

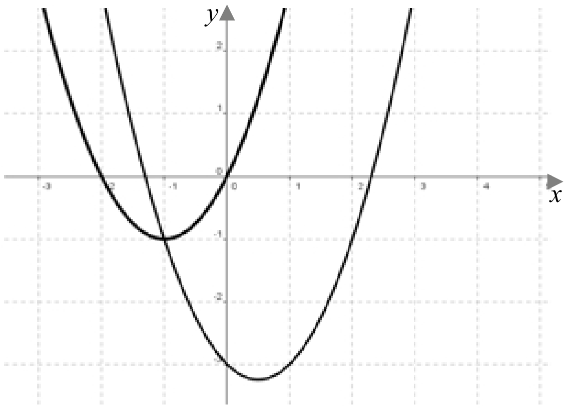
5. a) Solution 2		
The sum of the first n terms of the geometric progression is $S_n = \frac{32 \cdot \left(\frac{1}{128^n} - 1\right)}{\frac{1}{128} - 1}$, $S_n = \frac{2^{12}}{127} \cdot \left(1 - \frac{1}{128^n}\right)$.	1 point	
The sequence $\{S_n\}$ is strictly increasing (since the sequence $\left\{\frac{1}{128^n}\right\}$ is strictly decreasing),	1 point	
and for all n, $S_n < \frac{2^{12}}{127}$.	1 point	
$(S_n <) \frac{2^{12}}{127} = \frac{4096}{127} < 32.5$, so the statement is true.	1 point	
Total:	**4 points**	

5. b)		
$a_1 \cdot a_2 \cdot a_3 \cdots a_n =$ $= \frac{1}{128} \cdot \frac{32}{128} \cdot \frac{32^2}{128} \cdot \ldots \cdot \frac{32^{k-1}}{128} \cdot \ldots \cdot \frac{32^{n-1}}{128} =$	1 point	
$= \frac{32^{1+2+3+\ldots+n-1}}{128^n}$	1 point	
The exponent of the numerator is the sum of the first $n-1$ positive integers, which, in closed form, equals $\frac{n(n-1)}{2}$.	2 points	
Since $32 = 2^5$ and $128 = 2^7$, it follows that $\frac{32^{1+2+3+\ldots+n-1}}{128^n} = \frac{2^{5\cdot\frac{n(n-1)}{2}}}{2^{7n}}$.	1 point	
Since $2048 = 2^{11}$, the equation $\frac{2^{5\cdot\frac{n(n-1)}{2}}}{2^{7n}} = \left(2^{11}\right)^{3n}$ needs to be solved.	1 point	
Hence $2^{5\cdot\frac{n(n-1)}{2}} = 2^{7n} \cdot 2^{33n} = 2^{40n}$,	1 point	
Since the exponential function is one-to-one (strictly monotonic), it follows that	1 point	
$5 \cdot \frac{n(n-1)}{2} = 40n$.	1 point	
(*n* is a positive integer, so) we can divide by *n*: $\frac{5}{2}(n-1) = 40$.	1 point	
The only solution of this equation is $n = 17$.	1 point	
Therefore the only solution of the original equation is $n = 17$.	1 point	
Total:	**12 points**	

6. a) **Solution 1**		
The two parabolas have a common point on the x-axis if and only if the quadratic equations $x^2 + px + 1 = 0$ and $x^2 - x - p = 0$ have a common root.	1 point	
The common root is a solution of the equation $x^2 + px + 1 = x^2 - x - p$.	2 points	
Rearranged: $x(p+1) = -(p+1)$.	1 point	
If $p = -1$, then every real number is a solution of the equation, thus the two parabolas are identical. ($y = x^2 - x + 1$). Therefore this case is not accepted.	1 point	
If $p \neq -1$,	1 point	
then $x = -1$ follows. So $p = 2$.	1 point	
Thus the equations of the two parabolas are $y = x^2 + 2x + 1$ and $y = x^2 - x - 2$. (Their common point is the point (–1; 0).)	1 point	
Total:	**8 points**	

6. a) Solution 2		
The two parabolas have a common point on the x-axis if and only if the quadratic equations $x^2 + px + 1 = 0$ and $x^2 - x - p = 0$ have a common root.	1 point	
Let x_1 and x_2 denote the two (not necessarily different) roots of the equation $x^2 + px + 1 = 0$ and let x_1' and x_2' denote the roots of the equation $x^2 - x - p = 0$. Applying Viète's formulae:	1 point	
$\left.\begin{matrix} x_1 + x_2 = -p \\ x_1 \cdot x_2 = 1 \end{matrix}\right\}$ and $\left.\begin{matrix} x_1' + x_2' = 1 \\ x_1' \cdot x_2' = -p \end{matrix}\right\}$	1 point	
Case (1) The solutions of the two equations are the same numbers ($x_1 = x_1'$, and $x_2 = x_2'$). Then $p = -1$ follows. However, in that case the two equations belong to the same parabola $y = x^2 - x + 1$, which is against the condition of the problem.	1 point	
Case (2) If the two sets of solutions are not equal but have a common element, e.g. $x_1 = x_1'$. Then it follows from the second equations obtained from Viète's formulae that $x_1 \cdot x_2 = 1$ and $x_1 \cdot x_2' = -p$. Hence $x_2' = -px_2$. Therefore the first equations obtained from Viète's formulae are $x_1 + x_2 = -p$ and $x_1 - px_2 = 1$. By subtracting the corresponding sides of the two equations: $x_2(p+1) = -(p+1)$.	1 point	
If $p = -1$, then case (1) occurs.	1 point	
If $p \neq -1$, then $x_2 = -1$. In this case, $p = 2$.	1 point	
Thus the equations of the two parabolas are $y = x^2 + 2x + 1$ and $y = x^2 - x - 2$. (Their common point is the point (–1; 0).)	1 point	
Total:	**8 points**	

6. b)



A sketch of the parabolas of equations $y = x^2 + 2x$ and $y = x^2 - x - 3$.	2 points	*1 point for each correctly represented parabola.*
The first coordinate of the common point of the parabolas is –1.	1 point	
Consider the functions $f:[-1;0] \to \mathbf{R}, f(x) = x^2 + 2x$, and $g:[-1;0] \to \mathbf{R}, g(x) = x^2 - x - 3$. The area of the figure in question is $T = \int_{-1}^{0} f(x)dx - \int_{-1}^{0} g(x)dx = \int_{-1}^{0} (f(x) - g(x))dx$.	1 point	*The 5 points are also due if the candidate correctly integrates the individual functions and then correctly subtracts the results.*
$T = \int_{-1}^{0} (x^2 + 2x - (x^2 - x - 3))dx = \int_{-1}^{0} (3x + 3)dx =$	2 points	
$= \left[\frac{3x^2}{2} + 3x\right]_{-1}^{0} =$	1 point	
$= 0 - \left(\frac{3}{2} - 3\right) = \frac{3}{2}$	1 point	
Total:	**8 points**	

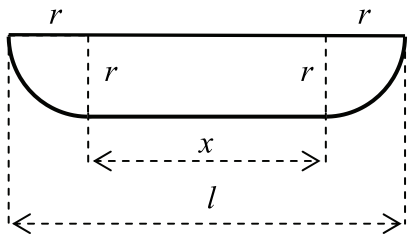
7. a)

1. false	1 point	*Award the points for clearly indicated correct answers only.*
2. false	1 point	
3. true	1 point	
4. true	1 point	
5. false	1 point	
Total:	**5 points**	

7. b)		
According to the statistics, the probability of a text message not being received is $\frac{1}{60}$, that is about 0.0167,	1 point	*These 2 points are also due if the idea is only reflected by the solution.*
thus the probability of the message being received by the addressee is 1–0.0167=0.9833.	1 point	
The probability of exactly 1 out of 3 text messages not being delivered is $\binom{3}{1}\cdot 0.9833^2\cdot 0.0167$,	1 point	*If the binomial coefficient is missing or wrong, award at most 1 of the last 2 points.*
which is approximately 0.0484 (that is 4.84%).	1 point	
Total:	**4 points**	*Calculated with $\frac{1}{60}$ and $\frac{59}{60}$, the result is $\frac{3481}{72000}\approx$ ≈ 0.0483.*

7. c)		
If n text messages are sent, the probability of all being delivered is 0.9833^n.	1 point	
Thus the probability of at least one of them not being delivered is $1-0.9833^n$.	1 point	*Award 2 points for the statement that the probability of all SMS being received is at most 2%.*
We want the smallest natural number n, such that $1-0.9833^n \geq 0.98$.	1 point	
Rearranged: $0.02 \geq 0.9833^n$.	1 point	
Hence $\log_{0.9833} 0.02 \leq n$ (since logarithm functions of bases less than 1 are strictly decreasing),	1 point	
$n \geq 232.3$.	1 point	*Calculated with* $\frac{1}{60}$ *and* $\frac{59}{60}$, *the solution* $n \geq 232{,}8$ *is obtained for the inequality.*
Therefore, if at least 233 text messages are sent, then the probability of at least one of them not being received is at least 0.98.	1 point	
Total:	**7 points**	*Award at most 4 points for part c) if the candidate solves an equation instead of an inequality but does not explain (e.g. by referring to monotonicity) why the value obtained is the smallest.*

8. a)



Let x denote the horizontal segment of the cross section of rounded edge. Then the width l of the channel is $l = 2r + x$. According to the conditions of the problem, $\frac{2r\pi}{2} + x = 20$	1 point	
and $\frac{r^2\pi}{2} + rx = 55$.	1 point	
From the first equation, $x = 20 - r\pi$. Substituted in the second equation: $\frac{r^2\pi}{2} + r(20 - r\pi) = 55$.	1 point	
$r^2\pi - 40r + 110 = 0$ $r_1 \approx 8.7$, but then $x_1 < 0$, so this is not a solution.	1 point	
$r_2 \approx 4.0$, hence $r = 4.0\,\text{cm}$.	1 point	
Thus $x = 20 - 4\pi = 7.434... \approx 7.4$. The width of the channel is $l = 2r + x \approx 8.0 + 7.4 = 15.4$ cm.	1 point	
Total:	**6 points**	*Take off 1 point only once if the answer is not rounded to one decimal place.*

8. b)		
According to the conditions of the problem, $r \cdot \pi + x = 20$, and the area $T = r \cdot x + \frac{r^2 \cdot \pi}{2}$ is a maximum. $(x = 20 - r\pi)$ The maximum of the function $T(r) = r\,(20 - r\pi) + \frac{r^2\pi}{2} = -\frac{r^2\pi}{2} + 20r$ ($0 < r \le \frac{20}{\pi}$) is needed.	1 point	
$T(r) = -\frac{r^2\pi}{2} + 20r$. By completing the square: $T(r) = -\frac{\pi}{2}\left(r - \frac{20}{\pi}\right)^2 + \frac{200}{\pi}$	2 points *	
(Since the first term of the sum is non-positive and the second term is a constant,) this sum will be a maximum where the first term is zero, that is, $r - \frac{20}{\pi} = 0$.	1 point*	
The maximum occurs at $r = \frac{20}{\pi}$.	1 point*	
Hence, because of $x = 20 - r\pi = 0$, the gutter of maximum capacity has a cross section of width $l = 2r + x = 2r$, so the statement is true.	1 point	
The cross section of the gutter of maximum capacity is a semicircle of radius $r = \frac{20}{\pi} \approx 6.4$ (cm).	1 point	
The task is to calculate the volume of a semi-cylinder of radius $r = \frac{20}{\pi} \approx 6.4$ cm and height $l = 250$ cm: $V = \frac{\left(\frac{20}{\pi}\right)^2 \pi \cdot 250}{2}$	1 point	*The correct answer of 16 litres is also obtained by calculating with r = 6.4:* $V = \frac{6.4^2 \cdot \pi \cdot 250}{2} \approx 16084.95$ (cm^3)
$V \approx 15915.5\ (\text{cm}^3)$	1 point	
≈ 16 litres	1 point	
Total:	**10 points**	

*Two alternative methods for the part marked with *:*

Method 2		
$r \mapsto -\frac{r^2\pi}{2} + 20r$ ($r \in \mathbf{R}^+$) is a quadratic function in which the leading coefficient ($-\frac{\pi}{2}$) is negative, that is, the function has a maximum.	1 point	
The two zeros of the function are $r_1 = 0$ and $r_2 = \frac{40}{\pi}$.	1 point	
The maximum occurs at the arithmetic mean of the zeros, at $r = \frac{20}{\pi}$.	1 point	
Since $0 < r \le \frac{20}{\pi}$, the function *T* in question also has its (global) maximum there.	1 point	

Method 3		
The derivative function of the function $r \mapsto -\frac{r^2\pi}{2} + 20r$ ($r \in \mathbf{R}^+$) is $r \mapsto -r\pi + 20$ ($r \in \mathbf{R}^+$).	1 point	
It is zero where $-r\pi + 20 = 0$, that is, there may be a maximum or minimum at $r = \frac{20}{\pi}$.	1 point	
The second derivative function $r \mapsto -\pi$ is negative at that point, so there is a maximum at $r = \frac{20}{\pi}$.	1 point	
Since $0 < r \le \frac{20}{\pi}$, the function *T* in question also has its (global) maximum there.	1 point	

9.		
Let a denote the average score of András in the first half (in the first five games) of the tournament. Then his total score in the first five rounds is $5a$.	2 points	
In the sixth, seventh, eighth and ninth games András scored a total of $23+14+11+20=68$ points.	1 point	
His average after the ninth round was $\frac{5a+68}{9}$.	1 point	
It is given that this average is greater than that after the first five games,	1 point	
that is $\frac{5a+68}{9}>a$.	1 point	
Hence $a<17$.	2 points	
Let x be the number of points scored by András in the tenth game. At the end of the tournament, the points average of András is $\frac{5a+68+x}{10}$.	2 points	
According to the given condition, $\frac{5a+68+x}{10}\geq 18$,	1 point	
that is $5a+68+x\geq 180$,	1 point	
therefore $x\geq 112-5a$.	1 point	
Since $a<17$, it follows that $x\geq 112-5a>112-5\cdot 17=112-85=27$.	2 points	
Hence $x>27$, so András must have scored at least 28 points in the tenth round.	1 point	
Total:	**16 points**	